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TOMOGRAPHIC DIAGNOSTICS OF TECHNICAL MATERIALS AND BIOLOGICAL TISSUES USING ELECTRIC CURRENT

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A mathematical model for impedance computer tomography methods is considered. The continuum formulation of the main problem is studied. Resolving integral equations are derived. A solution algorithm based on the Bubnov–Galerkin method with linearization of nonlinear resolving equations is developed. A numerical example is given, and numerical results are analyzed. Some drawbacks of the model are considered together with methods for avoiding them.

Introduction. Mathematical models of tomographic diagnostics, which are advanced methods for nondestructive quality control in technology and those of noninvasive biomedical diagnostics, reduce to inverse problems for ordinary differential equations [1, 2], partial differential equations [3], integral and, in some cases, algebraic systems of equations [3, 4], which are usually overdetermined.

Classification according to the type of obtainable and processed data is a rough classification of tomography methods and their mathematical models. These data can be related to material properties at a given point (Damadyan's point excitation of the nuclear magnetic resonance in NMR tomography [2]) or represent integrals over a set of paths inside the object under study (in x-ray, acoustic, emission, and NMR tomography), over its cross sections, or over its volume. The last two variants refer to impedance computer tomography (ICT).

The above classification is convenient for studying signal-reconstruction stability. In the case of point excitation or strictly monotonic growth or decay of the signal, depending on the amount of the substance or the intensity of the process under study at a given point (or in a given particle), the reconstruction is stable and the problem unequivocally solvable. However, point excitation studies are very time-consuming, which makes them inapplicable to studying extended or traveling objects. Therefore, in practice, for the most part, methods employing path integrals are used.

The problem of reconstruction of a function from a set of its path integrals (being predominantly straight lines) is known as the Radon problem. This problem has an exact solution [5].

In spite of the existence of the exact solution, various approximate methods are used in practice, because the numerical implementation of the Radon formula requires calculating too evolved singular integrals. With a proper sampling procedure, the process of approximate reconstruction is stable: in each particular problem, it is possible to select the number of measured integrals and choose such a sampling procedure that ensures a desired reconstruction accuracy.

In studying an object by the ICT method, the object experiences the action of an electric current distributed over a planar cross-section of the object or over its volume. The data gained in this case are the density of the electric current through the boundary or the distribution of the electric potential over the latter, which are obviously related by a certain integral relation, say, via the Green function [6]. The reconstruction algorithm here is less stable compared to problems dealing with known path integrals. A rough estimate of the stability of an approximate reconstruction can be obtained as follows.

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Let h be a typical sampling parameter, e.g., the diameter of finite elements or the step of a finite-difference grid. Then, the change of the parameter under reconstruction over distances of order h causes a change of order $O(h)$ in the path integral, of order $O(h^2)$ in the surface integral, and of order $O(h^3)$ in the volume integral. It follows from here that the problem of reconstructing a function from its volume integrals is the least stable and, hence, the most difficult one.

It should be noted that first theoretical estimates of stability in problems on reconstructing a function from its surface or volume integrals were obtained by Alessandrini [7].

The weak dependence of the measured parameters on local variations of the function to be reconstructed is the reason for a rather low resolving power of ICT-type methods. Simultaneously, the ICT method has a number of advantages over other tomography methods. First, with certain limitations on the amplitude of the electric current, the latter causes no damages to the biological tissue, whereas such damages cannot be avoided in x-ray or emission tomography. Second, the instrumentation used for implementing the ICT method is cheaper than that for x-ray, emission, and NMR tomographs. In addition, this instrumentation is small and can be made portable. Finally, the main parameter to be reconstructed, the electrical conductivity of the biological tissue, is sensitive to various pathologies caused by diseases and, in engineering, to harmful damages such as cracks.

In the present paper, the results of numerical experiments on electrical conductivity reconstruction by the Newton successive approximation method are described. These results may be helpful in improving the efficiency of ICT methods (in particular, their accuracy and resolving power).

1. Description of the Method. The essence of the method is as follows. Knowing the relation between the electric potential distribution over a surface and the measured distribution of the electric current density, we are going to find the distribution of the electric conductivity over the volume occupied by the medium under study. The model of [8], which gives an insight into the essence of the method, is related to representation of the volume conductivity in the form of an electric circuit consisting of resistors connected in series and in parallel. The circuit has N external terminals (electrodes), to which one may apply an electric potential and measure the induced currents; the measurements at internal points of the circuit are assumed impossible.

As applied to problems on reconstruction of elasticity moduli of an inhomogeneous medium, including problems on detecting internal defects, the discrete model can be constructed using the finite-element method, finite-difference method, or approximation of a three-dimensional structure with a rod system.

It is necessary to set a certain structure of the discrete system, say, in the form of resistors arranged in tetragons linked, in turn, to other tetragons at their vertices. We assign to each nodal point some coordinates of a corresponding point of the volume under study with respect to some preliminary chosen coordinate system, e.g., a Cartesian one.

We arrange the values of the potentials at the external terminals in a vector $\boldsymbol{\varphi}$ of dimension N, and designate the corresponding vector of currents as $\boldsymbol{j}$. With a certain basis chosen in the space of vectors $\{\boldsymbol{\varphi}\}$, for instance, with a Cartesian one

$$\boldsymbol{\varphi}^{(1)} = (1,\, 0,\, 0, \ldots, 0), \quad \boldsymbol{\varphi}^{(2)} = (0,\, 1,\, 0, \ldots, 0), \ldots, \quad \boldsymbol{\varphi}^{(N)} = (0,\, 0,\, 0, \ldots, 1),$$

performing experiments for all vectors of the basis (i.e., measuring the set of currents $\{\boldsymbol{j}^{(i)}\}_{i=1}^{N}$ through the electrodes), we can find the electric resistances of all resistors in the circuit. The latter formulation is known as the Telegen theorem.

Practical algorithms for determining the resistances are iterative in principle since the system of Kirchhoff equations being considered in the problem of interest as a system for the potentials and resistances of the resistors is nonlinear. One of the first algorithms for solving this problem was proposed by Dines and Lytles [8]. As applied to two-dimensional problems, the finite-element method was developed by Hua, Woo, et al. [9, 10]. Some estimates of the errors of solving inverse quasilinear problems were obtained by Karkkainen [11].

An algorithm constructed on the basis of the continual formulation of the problem is described below, and some regularization procedures for the problem are given. A more accurate formulation is also described, aimed at gaining a better insight into the problem, which uses more substantiated physical relations and more complicated mathematical models.

2. Continual Model. Let Ω be a region occupied by the object under study and $\partial\Omega \equiv \Sigma$ be the boundary of the region Ω. We designate the electric potential distributed over Ω as $\varphi = \varphi(\boldsymbol{x})$ $(\boldsymbol{x} \in \Omega)$ and the electric conductivity of the material in the region Ω as $\sigma = \sigma(\boldsymbol{x})$.

Assume that a mapping Q_σ is known, which defines the distribution of the current density $j_{\nu\Sigma}(\boldsymbol{x})$ across the surface Σ corresponding to each potential distribution $\varphi(\boldsymbol{x})|_\Sigma \equiv \varphi_\Sigma(\boldsymbol{x})$:

$$j_{\nu\Sigma} = \sigma(\nabla\varphi)\cdot\boldsymbol{\nu}\Big|_\Sigma.$$

Here $\varphi = \varphi(\boldsymbol{x})$ is the solution of the Dirichlet problem:

$$\nabla\cdot(\sigma\nabla\varphi) = 0, \qquad \boldsymbol{x}\in\Omega; \tag{1}$$

$$\varphi\Big|_\Sigma = \varphi_\Sigma(\boldsymbol{x}), \qquad \boldsymbol{x}\in\Sigma. \tag{2}$$

The formal definition of the operator Q_σ, given by Calderon, has the form

$$Q_\sigma(\varphi_\Sigma) = \int\limits_\Omega \sigma|\nabla\varphi|^2\,d\Omega = \int\limits_\Sigma \varphi_\Sigma(\sigma(\nabla\varphi_D\cdot\boldsymbol{\nu}))\Big|_\Sigma\,d\Sigma = \int\limits_\Sigma \varphi_N j_{\nu\Sigma}\,d\Sigma, \tag{3}$$

where φ_D is the solution of the Dirichlet problem with the boundary condition for the potential $\varphi\Big|_\Sigma = \varphi_\Sigma$ and φ_N is the solution of the Neumann problem with the boundary condition for the electric current $j_{\nu\Sigma}$.

The problem is formulated as follows: given the operator Q_σ, it is required to find the function $\sigma = \sigma(\boldsymbol{x})$ $(\boldsymbol{x}\in\Omega)$.

The existence and uniqueness of the solution were proved by Nachman [12]. The theory developed in [12] is based on the extension of the results of Gel'fand and Levitan [13] to the case of partial differential equations. The reduction to a problem of the Gel'fand–Levitan type can be made by the substitution $\varphi = \sigma^{-1/2}w$, which brings Eq. (1) to the form $-\Delta w + qw = 0$, where $q = \sigma^{-1/2}\Delta\sigma^{1/2}$. On finding the "spectral function" $q = q(\boldsymbol{x})$, the conductivity $\sigma = \sigma(\boldsymbol{x})$ can be reconstructed by solving the Dirichlet problem $-\Delta\sqrt{\sigma} + q\sqrt{\sigma} = 0$, $\sqrt{\sigma}\Big|_\Sigma = f(\boldsymbol{x})$, where the function $f(\boldsymbol{x})$ $(\boldsymbol{x}\in\Sigma)$ is calculated from the data at the boundary of the object under study for the surface potential and the current through the boundary.

It should be noted that Nachman [12] gives no effective algorithm for finding the solution since intermediate manipulations, which contain passages to limits in expressions that include single- and double-layer potentials for the operator $\nabla\cdot(\sigma\nabla)$, cannot be calculated in principle before determining the conductivity σ (only some general properties such as smoothness can be formulated).

Thus, the problem of developing algorithms for the inverse problem remains urgent. In Sec. 3, we give a solution to this problem under the assumption that $\varphi_\Sigma(\boldsymbol{x}) \in H^{1/2}(\Sigma)$. In this case, we have $\varphi(\boldsymbol{x}) \in H^1(\Omega)$.

3. Derivation of Resolving Equations and Some Their Properties. Since the space $H^{1/2}(\Sigma)$ is separable, a countable set that is dense everywhere can be chosen, denoted as $\{\varphi_\Sigma^{(i)}\}$. According to the formulation of the main problem, each function $\varphi_\Sigma^{(i)}$ may be put into correspondence to a unique function $j_{\nu\Sigma}^{(i)}$ having the meaning of the electric current through the boundary and determinable from the experiment. Taking into account definition (3), we conclude that the operator Q_σ is known: $\varphi_\Sigma \xrightarrow{Q_\sigma} j_{\nu\Sigma}$. Apparently, this operator is known only approximately, since the number of real experiments is limited and, hence, only a limited number of pairs $\{\varphi_\Sigma$ and $j_{\nu\Sigma}\}$ is available.

To reveal some general properties of the problem, we suppose, as in [12], that the relation between the solution of the Dirichlet problem $\varphi_D^{(i)}(\boldsymbol{x}_0)$ in the region Ω and the boundary condition $\varphi_\Sigma^{(i)}(\boldsymbol{x})$ $(\boldsymbol{x}\in\Sigma)$ is known, expressed in terms of the Green function $G_\sigma(\boldsymbol{x},\boldsymbol{x}_0)$:

$$\varphi_D^{(i)}(\boldsymbol{x}_0) = \int\limits_\Sigma \sigma(\boldsymbol{x})\,\frac{\partial G_\sigma(\boldsymbol{x},\boldsymbol{x}_0)}{\partial\nu_x}\,\varphi_\Sigma^{(i)}(\boldsymbol{x})\,d\Sigma. \tag{4}$$

Solution (4) corresponds to the current through the boundary $j_{\nu\Sigma}^{(i)}$; hence, to determine the conductivity in the region Ω, we use the following integral equations:

$$\sigma(\boldsymbol{x}_0)\,\frac{\partial}{\partial\nu_{x_0}}\int\limits_\Sigma \sigma(\boldsymbol{x})\,\frac{\partial G_\sigma(\boldsymbol{x},\boldsymbol{x}_0)}{\partial\nu_x}\,\varphi_\Sigma^{(i)}(\boldsymbol{x})\,d\Sigma = j_{\nu\Sigma}^{(i)} \quad \forall\, i. \tag{5}$$

The following properties of Eqs. (5) are worth noting.

1. The Green function G_σ depends on the prior unknown distribution of conductivity σ over the region Ω.

2. Each of Eqs. (5) may have nonunique solutions. This formulation readily follows from the following physical reasoning: for a fixed distribution of the boundary potential, identical electric currents can be obtained for different internal conductivity distributions.

3. The classical ill-posedness [1] is observed: small changes in the current through the boundary may correspond to large changes in the integrand in Eq. (5).

4. Successive Approximation Method. To develop the solution algorithm, we use a variational method and linearization. The variational approach is based on the Bubnov–Galerkin method. The superscript V denotes the functions that correspond to the true conductivity distribution $\sigma = \sigma^V$: $\varphi^{(i)} = \varphi^{(i)V}$ and $j^{(i)}_{\nu\sigma} = j^{(i)V}_{\nu\sigma}$. These functions are assumed to be known from the experiment. The iterative method for finding the function σ^V consists of several cycles, each containing the following steps.

Step 1. A zero-order approximation $\sigma^V \approx \sigma^{(0)}(\boldsymbol{x})$ $(\boldsymbol{x} \in \Omega)$ is specified.

Step 2. For each function $\varphi^{(i)}_\Sigma$, the Dirichlet problem with the boundary condition $\varphi^{(i,0)}\Big|_\Sigma = \varphi^{(i)}_\Sigma(\boldsymbol{x})$ $(\boldsymbol{x} \in \Sigma)$ is solved; afterwards, the boundary current distribution $j^{(i,0)}_{\nu\Sigma}$ is calculated.

Step 3. An arbitrary function $\chi \in H^1(\Omega)$ is chosen, and the difference, weighted with χ, between the true current distribution $j^{(i)V}_{\nu\Sigma}$ and the current $j^{(i,0)}_{\nu\Sigma}$ (weighted residual) is calculated:

$$\delta J^{(i,0)} = \int\limits_\Sigma (j^{(i)V}_{\nu\Sigma} - j^{(i,0)}_{\nu\Sigma})\chi \, d\Sigma.$$

Using the Gauss–Ostrogradskii formula, we find the following expressions for the residual $\delta J^{(i,0)}$:

$$\delta J^{(i,0)} = \int\limits_\Sigma \Big(\sigma^V \frac{\partial \varphi^{(i)V}}{\partial \nu} - \sigma^{(0)} \frac{\partial \varphi^{(i,0)}}{\partial \nu}\Big)\chi \, d\Sigma = \int\limits_\Omega (\sigma^V \nabla\varphi^{(i)V} \cdot \nabla\chi - \sigma^{(0)} \nabla\varphi^{(i,0)} \cdot \nabla\chi) \, d\Sigma. \tag{6}$$

Then, the differences $\delta\sigma = \sigma^V - \sigma^{(0)}$ and $\delta\varphi^{(i)} = \varphi^{(i)V} - \varphi^{(i,0)}$ are introduced (the superscript corresponding to the order of approximation at the residuals is omitted), by which the right sides of definition (6) are linearized:

$$\delta J^{(i,0)} \approx \int\limits_\Sigma \Big(\delta\sigma \frac{\partial \varphi^{(i,0)}}{\partial \nu} + \sigma^{(0)} \frac{\partial \delta\varphi^{(i)}}{\partial \nu}\Big)\chi \, d\Sigma \approx \int\limits_\Omega [\delta\sigma \nabla\varphi^{(i,0)} \cdot \nabla\chi + \sigma^{(0)} \nabla(\delta\varphi^{(i)}) \cdot \nabla]\chi \, d\Sigma. \tag{7}$$

Replacing in (7) the approximate equality by the exact one, we obtain two variational equations (the value of $\delta J^{(i,0)}$ is known), the second one relating two unknown functions $\delta\sigma$ and $\delta\varphi^{(i)}$ in the region Ω.

Step 4. Formally regarding the function $\delta\sigma$ as a known one, we may express all functions $\delta\varphi^{(i)}$ in terms of $\delta\sigma$. It follows from the second equation of (7) that this procedure is equivalent to the Dirichlet problems $\nabla \cdot [\sigma^{(0)} \nabla(\delta\varphi^{(i)})] = -\nabla \cdot [\delta\sigma \nabla(\varphi^{(i,0)})]$ and $\delta\varphi^{(i)}\Big|_\Sigma = 0$. The solutions found in this manner can be written as

$$\delta\varphi^{(i)} = L^{(i)}(\delta\sigma). \tag{8}$$

Obviously, the operators $L^{(i)}$ are linear.

Step 5. Solutions (8) are substituted into the first equation of (7):

$$\delta J^{(i,0)} = \int\limits_\Sigma \Big[\delta\sigma \frac{\partial \varphi^{(i,0)}}{\partial \nu} + \sigma^{(0)} \frac{\partial (L^{(i)}(\delta\sigma))}{\partial \nu}\Big]\chi \, d\Sigma. \tag{9}$$

It should be noted that system (9) is overdetermined; hence, in order to solve it, one has to apply special procedures, for instance, the least squares technique. This procedure is described below for a discrete system. The sampling is achieved by construction of a set of linearly independent functions χ_k and consecutive substitution of these functions into Eqs. (9). Preliminarily, the sought function $\delta\sigma$ is approximated by an expansion in a given basis. By these manipulations, the set of integral equations (9) is transformed into a set of linear algebraic systems

$$[R^{(i)}]\delta\boldsymbol{\sigma} = \delta\boldsymbol{J}^{(i)},$$

where $\delta\boldsymbol{\sigma}$ and $\delta\boldsymbol{J}^{(i)}$ are the vectors (columns) resulting from sampling the functions $\delta\sigma$ and $\delta J^{(i)}$. The solution obtained by the least squares method has the following form:

$$\delta\boldsymbol{\sigma} = ([R]^{\mathrm{t}}[R])^{-1}[R]^{\mathrm{t}}\delta\boldsymbol{J}.$$

Here $\delta \boldsymbol{J}$ is the vector-column composed from the vector-columns $\delta \boldsymbol{J}^{(i)}$ and $[R]$ is the matrix composed from the submatrices $[R^{(i)}]$, in line with the structure of the vector $\delta \boldsymbol{J}$; the superscript "t" denotes transposition.

Step 6. A higher-order approximation for the conductivity $\sigma^{(0)}$ is obtained using the formula

$$\sigma^{(1)} = \sigma^{(0)} + \beta \delta \sigma^{(0)}, \tag{10}$$

where the step β is an experimentally adjustable numeric parameter. Next, we pass on to step 2 of the above algorithm.

5. Numerical Example. The algorithm proposed in Sec. 4, was applied to a two-dimensional problem in a rectangular domain, which, after introducing dimensionless variables, was transformed to a unit square. A second-order finite-difference method with a five-node cross scheme was used for sampling. The nodal points were enumerated from left to right and from bottom to top.

Examination of numerical results allows the following conclusions.

1. If the boundary Neumann-type operator $\sigma \nabla \varphi \cdot \boldsymbol{\nu}$ is approximated, similarly to the operator $\nabla \cdot (\sigma \nabla \varphi)$, with second-order accuracy, then the algorithm described in Sec. 4 does not allow finding a solution even for a small number of degrees of freedom (about 20–30), since the matrix $[R]^{\mathrm{t}}[R]$ turns out to be almost degenerate; hence, regularization is necessary.

2. For the same number of unknown variables, a simplest regularization, replacement of the approximation of the boundary operator $\sigma \nabla \varphi \cdot \boldsymbol{\nu}$ of second-order accuracy with the first-order approximation (simpler in implementation) is quite efficient. After 300 iterations, the reconstruction error for even piecewise-constant conductivity distributions σ^V was of the order of 10^{-4}. The same result was obtained after 100 iterations after introducing the constraint

$$0 < \sigma_{\min} = \mathrm{const} \leqslant \sigma^{(k)} \leqslant \sigma_{\max} = \mathrm{const} \tag{11}$$

(k is the iteration number), which makes allowance for the physical properties of actual materials.

It should be noted that Dines and Lytles [8] who reconstructed 25 parameters solved the problem without applying a regularization procedure. It seems that approximation of the electric-circuit resistance by the Kirchhoff laws and the approximation of the boundary current were of different orders in that study.

3. As the number of the parameters to be found increases to 100–200 of them, the above simplest regularization proved to be inefficient even with making allowance for constraint (11). After introducing the first-order regularization (addition of a combination of squared first-order derivatives of the solution to the problem functional) [1] with the simultaneous use of constraints (11), we managed to find an appropriate solution by varying three parameters in the iteration process: the step length β in formula (10), the parameter ε_0 of the zero-order regularization, and the parameter ε_1 of the first-order regularization.

Some results obtained for the conductivity distribution

$$\sigma^V(x, y) = \begin{cases} 2, & 0 \leqslant y \leqslant 0.5, \\ 1, & 0.5 < y \leqslant 1 \end{cases}$$

are plotted in Figs. 1 and 2. Figure 1a shows the solution after 100 iterations (the node numbers and the values of the solution at the nodes are plotted along the abscissa and ordinate axes, respectively). In the vicinity of the jump in the conductivity σ^V, some oscillations of the solution are observed; however, the approximate solution outside this region is close to the exact solution.

Figure 1b and Fig. 1c show the solutions after 300 and 500 iterations, respectively. It is seen that, as the total number of iterations increases, the accuracy of the solutions improves; however, the oscillations in the vicinity of the jump are still observable even after 500 iterations.

Figure 2 illustrates the behavior of the largest (in the absolute value) components s_i of the vector $\delta \boldsymbol{\sigma}$ as a function of the total number of iterations i. The nonmonotonic behavior of this curve can be explained by the fact that the parameters β, ε_0, and ε_1 were changed after a certain number of iterations.

Conclusions. In biomedical diagnostics problems, the model given by relations (1) and (2) may appear to be too crude. For composite materials sensitive to the loading rate, displaying memory, etc., the model can be constructed using the Maxwell equations and the general constitutive relations

$$\boldsymbol{D}(t) = \int\limits_{-\infty}^{t} \int\limits_{\Omega} \hat{\varepsilon}(t, t', \boldsymbol{x}, \boldsymbol{x}') \cdot \boldsymbol{E}(t', \boldsymbol{x}')\, d\boldsymbol{x}'\, dt', \tag{12}$$

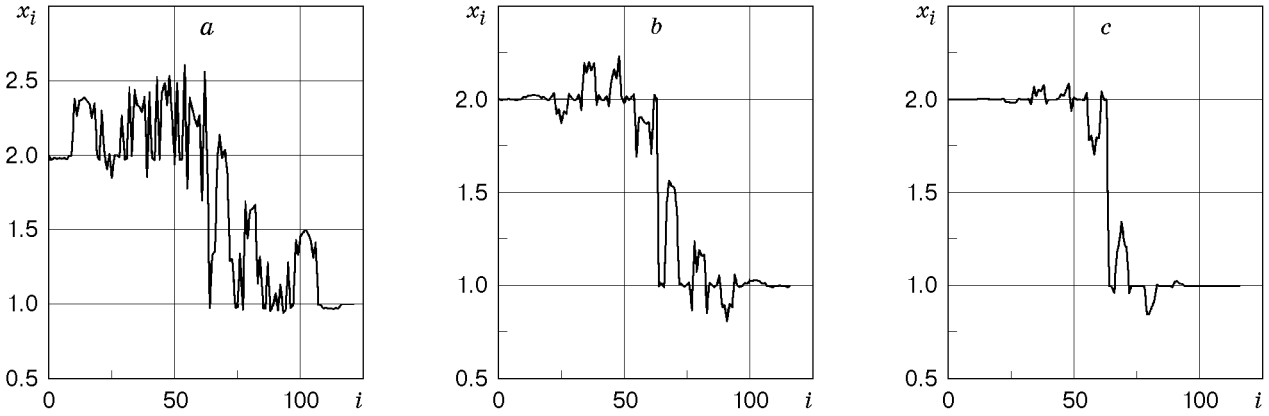

Fig. 1. Solutions of the problem after 100 (a), 300 (b), and 500 iterations (c).

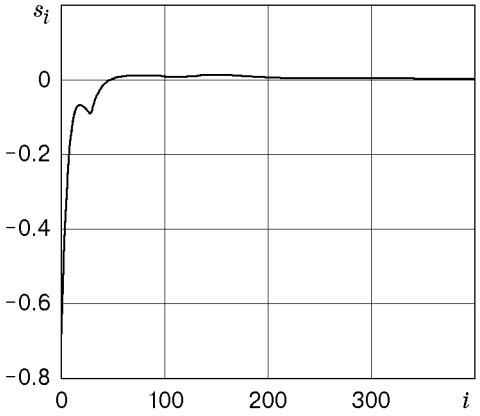

Fig. 2. Behavior of the correcting additive.

where $\hat{\varepsilon}(t, t', \boldsymbol{x}, \boldsymbol{x}')$ is the permittivity tensor, $\boldsymbol{E}$ is the electric field vector, and $\boldsymbol{D}$ is the induction vector. Relation (12) allows one to take into account such phenomena as nonlocality in time or space, anisotropy, or transient processes.

The contact electrodes used in biomedical diagnostics for exerting an influence on a biological object and for measurements have finite dimensions. Because of this, in practice the researcher deals with a certain distribution of the electric potential in the contact region Σ_E^i ($i = 1, 2, \ldots, N_E$, where N_E is the total number of electrodes) rather than with a point contact between the electrode and the object: $\varphi\Big|_{\Sigma_E^i} = \varphi_\Sigma^{(i)}(\boldsymbol{x})$ ($\boldsymbol{x} \in \Sigma_E^i \subset \Sigma$). Since the electric current through the boundary is zero outside these regions, we have $\sigma(\nabla\varphi)\cdot\boldsymbol{\nu}\Big|_\Sigma = 0$ and $\boldsymbol{x} \in \Sigma \setminus \bigcup\limits_i \Sigma_E^i \equiv \Sigma_j$. Hence, the mixed problem $\varphi\Big|_{\Sigma\setminus\Sigma_j} = \varphi_\Sigma^{(i)}$, $\sigma(\nabla\varphi)\cdot\boldsymbol{\nu}\Big|_{\Sigma_j} = 0$ and not the Dirichlet problem (or the Neumann problem, in the dual formulation [6]) should be the main problem in the reconstruction algorithm. This conclusion was verified by direct processing of experimental data obtained for specimens made of electrically conducting paper. It turned out that the difference between the calculated and experimental data could be as large as 200–300%.

The problem considered above is similar to that for reconstruction of elastic moduli of a spatially nonuniform elastic medium [6, 14, 15]. The results obtained indicate that, for achieving a desired accuracy in elasticity theory problems, it is necessary, first, to apply a first- or higher-order regularization procedure and, second, the total number of iterations should be several hundreds or even thousands even with an optimal choice of regularization parameters and the step length.

It should be also noted that quasistatic experiments may prove inadequate for reconstructing distributions of moduli of an anisotropic medium [15]; hence, results of dynamic tests should be invoked.

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